December 1, 1993 WOCOMAL VARSITY MEET

ROUND I: Arithmetic - percent, interest, discount, fractions and decimals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The product of the repeating decimals $0.3333\cdots$ and $0.2222\cdots$ is a repeating decimal. Find the second digit to the right of the decimal point in that product.

2. The harvest was cornucopian, as it was 120% greater than last year's. If the yield was 132,000 bushels, how many bushels were harvested last year?

3. A store gave a discount on a number of items. Two days later it gave a 20% discount on top of the first discount. If the two successive discounts are equivalent to a single 34.4% discount, what was the % of the first discount?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____ %

Algonquin, Auburn, South

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ROUND II: Set theory and logic

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. How many subsets of $\{G, O, L, F\}$ have G as an element?

2. Nine people are standing in a circle. Juan begins counting with "one." Tony, beside him, says "two" and the remaining people continue around the circle counting "three," "four," "one," "two," "three," "four.", etc. Each person who calls out "four" leaves the circle and the counting continues. How many times will Tony call out a number other than "four"?

3. If A, B, and C are subsets of $U = \{0, 1, 2, 3, 4, 5, 6\}$ and $A \cup B = \{1, 2, 3, 4, 5\}$, $A \cup C = \{1, 3, 4, 5\}$, $A \cap C = A$, $A \cap B = \phi$, and $B \cap C = \{3, 5\}$, specify B by listing its elements.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. $B = \{$ _____ $\}$

Burncoat, Doherty, St. Peter-Marian

December 1, 1993 WOCOMAL VARSITY MEET

ROUND III: Algebra 1 – open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1 Solve $\begin{cases} 6x + 2y = 11 \\ 3x - 7y = 1 \end{cases}$

2. If $\sqrt[5]{x} = 4$, what is the value of $\sqrt{x}$?

3 A train averages q miles per hour for s hours followed by an average of r miles per hour for the next t hours. What is its average speed during those $s + t$ hours?

ANSWERS

(1 pt) 1. $x =$ _____, $y =$ _____

(2 pts) 2. _____

(3 pts) 3. _____

Bartlett, Tahanto, Worcester Academy

December 1, 1993 WOCOMAL VARSITY MEET

ROUND IV: Sequences and series

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1 If the first term of an arithmetic sequence is 6 and the tenth term is 12, find the common difference.

2 In a sequence, $a_n = 3a_{n-1} - 1$ and $a_5 = 5$.
Find term a_1

3. In the sequence 6, x, y, 16, the first three terms form an arithmetic sequence and the last three terms form a geometric sequence. Find all possible ordered pairs (x, y) for which this happens.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

December 1, 1993 WORCESTER VARSITY MEET

ROUND V: Matrix and determinant operations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve for x

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

2. Find matrix B if

$$\begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} \cdot B = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -10 & 11 \end{bmatrix}$$

3. State a condition expressing a in terms of b and c which is necessary for the value of the determinant

$$\begin{vmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

to be zero.

ANSWERS

(1 pt) 1. $x =$ _____

(2 pts) 2. $\begin{bmatrix} \quad \end{bmatrix}$

(3 pts) 3. _____

Burncoat, Doherty, Worcester Academy

December 1, 1993 WOCOMAL VARSITY MEET

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM
AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1. Johnny had a fielding average of .925. After four errors in five chances, his average dropped to .896. Including these four, how many errors had he made all season?

2. Six switches, A, B, C, D, E, F are connected on an electrical circuit so that a light will go on if and only if at least one switch is closed. How many arrangements of open and closed switches exist that will permit the light to go on? (Consider switch A closed and all the rest open to be different from switch B closed and all the rest open.)

3. Solve $\dfrac{3}{x^2-5x+6} - \dfrac{x}{x-3} = \dfrac{2}{x-2}$.

4. The digital sum of a number is the sum of its digits. If the resulting sum is greater than 9, the digits are summed again until a one digit number is obtained. If the digital sum of an integer n is denoted $d(n)$, evaluate $\sum_{n=1}^{100} d(n)$.

5. If $\begin{bmatrix} -2 & 0 & 1 \\ 3 & 3 & 3 \\ 3 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ -6 \end{bmatrix}$, find $x+y+z$

6. Find all possible pairs of positive integers (A, B) so that $\dfrac{A}{11} + \dfrac{B}{3} = \dfrac{31}{33}$

7. Find the middle term in the expansion of $\left(\dfrac{2}{x} + \dfrac{x}{2}\right)^8$.

8. Find the ratio of the side of a square to the side of an equilateral triangle if the areas of each are equal.

9. Find the area of the region bounded by $|x+y| + |x-y| = 2$ in the xy plane.

Algonquin, Auburn, Marlboro, Shepherd Hill, Tahanto, West Bolyston, Worcester Academy

1, 1993

1. 7
2. 60,000
3. 18%

1 8
2 6
3 $\{2, 3, 5\}$

1. $x = \frac{5}{3}$ $y = \frac{1}{2}$
2. 32
3 $\frac{8s+rt}{s+t}$

1. $\frac{2}{3}$
2 $\frac{5}{9}$
3. $(1,-4), (9,12)$ need both

1. -2
2. $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \end{bmatrix}$
3. $a = 2b - c$

1. 13
2. 63
3 -3
4. 496
5. 5
6. $(3,2)$
7. 70
8. $\frac{\sqrt[4]{3}}{2}$ or $\sqrt[4]{3}:2$
9. 4